Outline

• Participant and Instructor Intro’s
• Discussion: What decisions do we want to make using a Reliability Analysis?
• The Math to enable good RAM decisions
• Some real world examples solving tough reliability related problems, making great RAM decisions
Participant Introductions

• Name
• Company
• Your role in reliability
• What you hope to gain from today’s tutorial
Your Presenter

• Over 40 years’ experience in Systems Engineering and Project Management
• Former Chair, INCOSE Risk Management Working Group
• INCOSE Technical Leadership Team, Former Assistant Director for Systems Processes
• IEEE AC Session Organizer: Risk Management and Lessons Learned (papers welcome!)
• Professor, Systems Engineering and RAM Engineering
  • Stevens Institute of Technology
  • University of Houston Clear Lake
  • University of Idaho
• Contact information at the end of tutorial
• If you have data and a challenging problem, we can coauthor a paper for next year’s conference.
• Link for slides at the end of tutorial, or contact me

Tutorial: Bayesian Reliability Analysis, What’s All the Fuss? - IEEE AC 2014
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Discussion

• Why do you do reliability analyses?
• What questions do you hope to use them to answer?
• What decisions will be made based on a reliability analysis?
Example Question for the Tutorial

- New vehicle – the Zeus 5000 SUV
- Plan to manufacture 100,000
- Want to offer 100,000 mile drivetrain warranty
- How much do we need to add to the price of each Zeus 5000 SUV to cover warranty costs, to make a profit?
  - Number that fail before 100,000 miles and repair costs determines how much total warranty policy will cost
  - Total warranty policy costs determines how much we need to add to the price of each Zeus 5000 SUV
- Answer based on Zeus 5000 SUV reliability at 100,000 miles
- We do have Zeus 5000 SUV test data, failures and survivors!
The Problem at the Root

- We are uncertain as to when a system or product will fail.
- Therefore, there will be an uncertainty distribution or model (denoted $x \sim pd(x)$) for when the system or product will fail.

$$t_f \sim pd_f(t_f)$$

- There may be no way to ever know this uncertainty distribution exactly.
  - Could in theory do an extremely thorough FMEA and derive from the physics everything that could cause a failure.
  - Rarely can anticipate everything, nor anticipate all possible conditions in which a system or product will be used.
  - In practice, never known exactly – this failure uncertainty model remains uncertain.
But, If We Knew that Failure Uncertainty Model …

• Then, we could calculate the risk that the system would fail before $T$.

\[
Risk(t_f \leq T) = P(t_f \leq T) = \int_0^T p d_f(\tau) d\tau
\]

• And, we could calculate the reliability at $T$

\[
R(T) = 1 - Risk(t_f \leq T) = P(t_f > T) = \int_T^\infty p d_f(\tau) d\tau
\]

• We could then answer all those important questions.
But We Don’t Know the Failure Uncertainty Model

• This means we are also uncertain about the Reliability at $T$.
• So there must be an uncertainty distribution model for the Reliability at $T$.

$$R(T) \sim pd_{R(T)} \left( R(T) \right)$$
$$\sim pd_{R(T)} \left( \int_{T}^{\infty} pd_{f}(\tau) d\tau \right)$$
But, If we Knew the Reliability Uncertainty Model ...

- We could calculate the risk that the Reliability at $T$ might be less than some critical Reliability, $R_c(T)$.

$$\text{Risk} \left( R(T) \leq R_c(T) \right) = P \left( R(T) \leq R_c(T) \right) = \int_0^{R_c(T)} p_d(\rho) d\rho$$

where $\rho \equiv R(T) = \int_T^{\infty} p_d f(\tau) d\tau$

- By calculating this Reliability risk, we can still make some very good decisions.
Well, What Do We Really Know for Certain?

• Our *data*, when we get it. These things actually happened or will have happened.
  
  
  • Failures – times when the system failed
  • Survivors – times of operation when the system had yet to fail
  • Other data types perhaps

• We are still uncertain about $pdf(t_f)$ and $pd_{R(T)}(R(T))$ though.
We can Reduce the Uncertainty about $pd_f(t_f)$

- We can do engineering analyses and derive from the physics a proper form for a failure uncertainty model (or pick a good one), and validate that form.
- Now, if we do good engineering, physics, math, and validation, we become certain about the proper form for $pd_f(t_f)$.
- The failure uncertainty model form will have parameters, $\Theta$, constants that determine the ultimate shape of $pd_f(t_f | \Theta)$.
- But, now we are uncertain about both $t_f$ and the values of the parameters $\Theta$; so this means we have a $pd(t_f, \Theta)$.

$$pd(t_f, \Theta) = \underbrace{pd_f(t_f | \Theta)}_{\text{form, validated and certain}} \underbrace{pd(\Theta)}_{\text{uncertain}}$$

4th Probability Axiom

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Substituting this into our Reliability Risk Equation

- We get functions of just the parameters, $\Theta$

$$\rho (\Theta) = R(T \mid \Theta) = \int_T^\infty pd(\tau, \Theta) d\tau = \int_T^\infty pd_f(\tau \mid \Theta) pd_\Theta(\Theta) d\tau$$

$$Risk \left( R(T) \leq R_c(T) \mid \Theta \right) = \int_0^{R_c(T)} pd_\rho \left( \rho (\Theta) \right) d\rho$$

- Now, we are uncertain only about $\Theta$, via $pd_\Theta(\Theta)$.
- If we could eliminate the uncertainty about $\Theta$ from our equations, then we would know our reliability risk.
- And make the best RAM decisions possible!
Killing Two Birds with One Stone

• Before we place the product in service, or test it, we are uncertain about the data we will get, and \( \Theta \).
• We can do some math.

\[
P(\Theta, \text{data}) = P(\Theta | \text{data}) P(\text{data}) = P(\text{data} | \Theta) P(\Theta) \quad \text{4th Probability Axiom}
\]

with algebra

\[
\Rightarrow P(\Theta | \text{data}) = \frac{P(\text{data} | \Theta) P(\Theta)}{P(\text{data})} \propto P(\text{data} | \Theta) P(\Theta)
\]

\[
\Rightarrow pd_\Theta(\Theta | \text{data}) \propto pd_f(\text{data} | \Theta) pd(\Theta) \quad \text{just a constant! L}^2 \text{ vector space property}
\]

Likelihood, from MLE

• Now, we have to deal with \( pd(\Theta) \).
• (The birds are still twittering weakly, but the product is going through testing; we will get data.)
Let’s Deal with \( pd(\Theta) \)

- \( pd(\Theta) \) is the marginal uncertainty model for \( \Theta \).
- Impossible to say what this marginal uncertainty model is, or should be.
- Best to be as objective as possible selecting \( pd(\Theta) \), to avoid assumptions tainting decisions.
- Methods exist to find the \( pd(\Theta) \) that maximize objectivity for the form of any particular \( pd_f(t_f|\Theta) \).
  - Derivable using three independent methods.
  - Virtually always get the same \( pd_{\text{obj}}(\Theta) \) for the form of \( pd_f(t_f|\Theta) \), regardless of the method used to derive.
  - Tabulated in references – validate the form of \( pd_f(t_f|\Theta) \), and simply look \( pd_{\text{obj}}(\Theta) \) up, and now substituting

\[
\Rightarrow pd_\Theta(\Theta|\text{data}) \propto pd_f(\text{data} | \Theta) \cdot pd_{\text{obj}}(\Theta)
\]
Now We Obtain our data, and Eliminate \( \Theta \)

- Substituting into \( \rho(\Theta) \), now that we have our data
  
  \[
  \rho(\Theta | \text{data}) = \int_T^\infty \rho_f(\tau | \Theta) \rho_{\Theta}(\Theta | \text{data}) d\tau \propto \int_T^\infty \rho_f(\tau | \Theta) \rho_f(\text{data} | \Theta) \rho_{\text{obj}}(\Theta) d\tau
  \]

  **Rearrange the integral to recognize \( \rho(\Theta) \)**

  \[
  = \left\{ \int_T^\infty \rho_f(\tau | \Theta) d\tau \right\} \rho_f(\text{data} | \Theta) \rho_{\text{obj}}(\Theta) = \rho(\Theta) \rho_f(\text{data} | \Theta) \rho_{\text{obj}}(\Theta)
  \]

- We can transform the Reliability at \( T \) as a function of \( \Theta \) into the Reliability at \( T \) as a function of the data.

  \[
  \rho(\text{data}) = \int_\Theta \cdots \int_\Theta \rho(\Theta) \rho_f(\text{data} | \Theta) \rho_{\text{obj}}(\Theta) d\Theta
  \]

  **kernel, \( K(\Theta, \text{data}) \)**

- This is an integral transform (remember Laplace and Fourier transforms, same sort of thing, different kernel!)

- We transform from our uncertain \( \Theta \) space into our known for certain data space!
Substituting $\rho(data)$ into our Reliability Risk Equation

\[
Risk\left(R(T) \leq R_c(T) \mid data\right) = P\left(R(T) \leq R_c(T) \mid data\right) = \int_0^{R_c(T)} pd_\rho\left(\rho(data)\right) d\rho
\]

\[
= \int_0^{R_c(T)} pd_\rho \left( \int_\Theta \cdots \int_\Theta \rho(\Theta) pd_f(data \mid \Theta) pd_{obj}(\Theta) d\Theta \right) d\rho
\]

\[
= \int_0^{R_c(T)} pd_\rho \left( \int_\Theta \cdots \int_\Theta \int_T^{\infty} pd_f(\tau \mid \Theta) d\tau \right) pd_f(data \mid \Theta) pd_{obj}(\Theta) d\Theta \right) d\rho
\]

\[
= f(data)
\]

Our Reliability Risk is now purely a function of what we know for certain, our data. (Both birds are dead!)
Our Zeus 5000 SUV data is Failures and Survivors

- \(pd_f(data|\Theta)\) is the familiar likelihood function
- In terms of failures \(t_{fi}\) and survivors \(t_{sj}\), it is

\[
pd_f(data|\Theta) = \prod_{i=1}^{n_f} pd_f(t_{fi}|\Theta) \times \prod_{j=1}^{n_s} \int_{t_{sj}}^{\infty} pd_f(\zeta|\Theta)d\zeta
\]

Just the Product of the densities at the failures
Just the Product of the reliabilities at the survivors

- What about \(pd_{obj}(\Theta)\)?
  - Look up the objective models for \(\Theta\) for the form of \(pd_f(t_f|\Theta)\), usually tabulated in a reference
  - Or, derive them if you have to
The Reliability Risk for the Zeus 5000 SUV

Substituting this likelihood into our equation from slide 17

\[ P(R(T) < R_c(T)|\text{data}) = \int_0^{R_c(T)} p_d(\rho|\text{data}) d\rho \]

\[
\propto \int_0^{R_c(T)} p_d(\rho) \left( \prod_{\Theta} \int_{t_{f_i}}^{t_{s_j}} \int_{T}^{\infty} \int_{t_{f_i}}^{t_{s_j}} \int_{t_{f_i}}^{t_{s_j}} \int_{t_{f_i}}^{t_{s_j}} \int_{t_{f_i}}^{t_{s_j}} p_d(\tau|\Theta) d\tau \prod_{i=1}^{n_f} p_d(f_i|\Theta) \prod_{j=1}^{n_s} p_d(f_j|\Theta) d\zeta \right) p_d(\Theta) d\Theta d\rho 
\]

the kernel, \( K(\Theta, t_{f_i}, t_{s_j}) \)

\[
= \int_0^{R_c(T)} p_d(\rho) \left( \prod_{\Theta} \int_{t_{f_i}}^{t_{s_j}} \int_{t_{f_i}}^{t_{s_j}} \int_{t_{f_i}}^{t_{s_j}} \int_{t_{f_i}}^{t_{s_j}} \int_{t_{f_i}}^{t_{s_j}} \int_{t_{f_i}}^{t_{s_j}} p_d(\tau|\Theta) d\tau \prod_{i=1}^{n_f} p_d(f_i|\Theta) \prod_{j=1}^{n_s} p_d(f_j|\Theta) d\zeta \right) p_d(\Theta) d\Theta d\rho 
\]

the kernel, \( K(\Theta, t_{f_i}, t_{s_j}) \)

\[
= f\left( \{t_{f_1}, t_{f_2}, \ldots, t_{f_{n_f}}\}, \{t_{s_1}, t_{s_2}, \ldots, t_{s_{n_s}}\} \right)
\]

Our expression is starting to get a little busy!

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We Validate a Weibull Model for the Zeus 5000 $pd_f(t_f|\Theta)$

- $\Theta$ for the Weibull failure model is two parameters, $\eta$ and $\beta$, so $pd_f(t_f|\Theta) \equiv pd_f(t_f|\eta,\beta)$.

Do the Weibull Substitutions, Eqn. Slide 19

$$P\left(R(T) < R_c(T) | \text{data} \right) = \int_0^{R_c(T)} \int_0^\infty \int_0^\infty \rho \left( \int_0^\infty \int_0^\infty e^{-\left(\frac{t}{\eta}\right)^\beta} \prod_{i=1}^N \left( \frac{\beta}{\eta} \right) \left( \frac{t_{fi}}{\eta} \right)^{\beta-1} e^{-\left(\frac{t_{fi}}{\eta}\right)^\beta} \prod_{j=1}^M e^{-\left(\frac{t_{sj}}{\eta}\right)^\beta} \left( \frac{1}{\eta} \right) \left( \frac{1}{\beta} \right) d\eta d\beta \right) d\rho$$

The Zeus 5000 SUV Weibull Kernel

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Trouble in Paradise

- Nobody can obtain *closed form solutions* for the integrals on:
  - Slide 17 – the Reliability Risk for general *data*
  - Slide 19 – the Reliability Risk when the *data are failures* and *survivors*
  - Slide 20 – the Reliability Risk when the *data are failures* and *survivors* and the Weibull model is valid
- Nobody can *approximate* (using ordinary numerical methods) these integrals either.
- But we can solve them very easily using an alternate numerical method, Markov chain Monte Carlo (MCMC).
An Aside:
Markov chain Monte Carlo

- Monte Carlo is a numerical method that approximates definite integrals and integral transforms.
  - Requires sampling of the integrand over the domain
  - Numerical method errors (errors in the integral approximation) grow smaller as the numbers of samples get larger.
- Markov chain Monte Carlo uses a Markov chain to sample the integrand, otherwise just Monte Carlo
  - A Markov chain can sample any integrand.
  - Very easy codes, but the Markov chains require manual tuning (cannot build a generic MCMC plug and chug tool)
  - Can sample high dimensional integrands: \( \Theta = \{\Theta_1, \Theta_2, \ldots, \Theta_n\} \)
  - Lots of MCMC references (end of presentation)
  - To develop good skills with MCMC usually requires some mentoring and/or classwork, tuning can be tricky
This Sounds Really Hard, But It's Amazingly Easy!

- Four Simple Steps to make any RAM decision objectively:

  - **Step 1:** Select and validate a failure uncertainty model, formulate the likelihood function for your **data**, and look up objective models.
    - You should always validate your failure uncertainty model anyway.
    - Easy to write out the likelihood function for most data types, some types may be challenging, but it is doable.

\[
\text{Risk}\left(R(T) \leq R_c(T) | \text{data}\right) = P\left(R(T) \leq R_c(T) | \text{data}\right) = \int_0^{R_c(T)} \text{pd}_\rho \left( \int \cdots \int_{T} \int_{\Theta} \text{pd}_f (\tau | \Theta) d\tau \left[ \text{pd}_f (\text{data} | \Theta) \text{pd}_{obj} (\Theta) d\Theta \right] d\rho \right)
\]
Easy Step 2

• **Step 2**: Use Markov chain Monte Carlo and obtain $N$ joint samples of $\Theta$ using the Kernel in Step 1.
  • Once you get the hang of MCMC, this gets easy.
  • Don’t have time to get a cup of coffee before an MCMC completes, even for a million samples
  • Every problem is different, and requires a different coding of the kernel and MCMC, but if you solve the same type of problem repeatedly, you can use the same codes.
Easy Step 3

- **Step 3**: Evaluate the Reliability at $T$ for the failure model at these $N$ joint samples of $\Theta_i$, integral transforming them into $N$ samples of the Reliability uncertainty model.
  - This is one line of code easy.
  - And, superfast.

\[
\text{Risk}\left( R(T) \leq R_c(T) \mid \text{data} \right) = P\left( R(T) \leq R_c(T) \mid \text{data} \right)
= \int_{0}^{R_c(T)} pd_\rho \left( \prod \cdots \prod T f (\tau \mid \Theta) \text{d}\tau \right) pd_f (\text{data} \mid \Theta) pd_{\text{obj}} (\Theta) \text{d}\Theta \text{d}\rho
\]
**Easy Step 4**

- **Step 4:** Compute the Risk integral by counting the number of $R_i(T)$ samples less than $R_c(T)$, and dividing by $N$
  - This may take three lines of code, but they are very simple.
  - This runs superfast, even for a million samples.

\[
\text{Risk} \left( R(T) \leq R_c(T) \mid data \right) = P \left( R(T) \leq R_c(T) \mid data \right) = \int_{0}^{R_c(T)} pd_{\rho} \left( \prod \prod \cdots \prod_{T} pd_{f} \left( \tau \mid \Theta \right) d\tau \right) pd_{f} \left( data \mid \Theta \right) pd_{obj} \left( \Theta \right) d\Theta \right) d\rho
\]
Revisiting Step 3, the Integral Transformation

- For your RAM decision, there may be some other quantity needed to make the decision, other than Reliability, or it may be some complex function of the Reliability.
- There is always some way to express this other quantity as a function of the failure uncertainty model parameters $Q = f(\Theta)$.
- To get the $N$ samples of $Q_i$, simply evaluate $f(\Theta_i)$.
- Then do an analogous form of Step 4 to calculate whatever risk you need to make the decision.
For the Zeus 5000 SUV: Failures, Survivors, Weibull

- **Step 1:** We validated that the Zeus 5000 SUV has but a single failure mode, so the Weibull is an ideal model to use. Our data is failures and survivors. Here is the kernel we code up.

\[
\prod_{i=1}^{N} \left( \frac{\beta}{\eta} \left( \frac{t_{fi}}{\eta} \right)^{\beta-1} e^{-\left(\frac{t_{fi}}{\eta}\right)^{\beta}} \right) \left[ \prod_{j=1}^{M} e^{-\left(\frac{t_{sj}}{\eta}\right)^{\beta}} \right] \left( \frac{1}{\eta} \right) \left( \frac{1}{\beta} \right)
\]

- **Step 2:** Use MCMC to obtain \( N \) joint samples of \( \eta \) and \( \beta \) using this kernel code.

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For the Zeus 5000 SUV Reliability Analysis

• **Step 3**: Evaluate the Reliability Equation at the $N$ joint samples of $\eta_i$ and $\beta_i$ transforms them into $N$ samples of $pd_R(\rho|data)$,

\[
R_i(T|data) = e^{-\left(\frac{T}{\eta_i}\right)^{\beta_i}}
\]

each simply

• **Step 4**: This does the final outside Risk integral.

\[
P\left(R(T) < R_c(T)|data\right) = \int_0^{R_c(T)} pd_\rho(\rho|data) d\rho \approx \sum_{i=1}^{N} \left\{\begin{array}{ll}
1 & R_i(T)|data < R_c(T) \\
0 & R_i(T)|data \geq R_c(T)
\end{array}\right\} \frac{1}{N}
\]
Why the Title?
Bayesian Reliability Analysis

• On Slide 14, we derived Bayes’ Law.

\[
P(\Theta, \text{data}) = P(\Theta | \text{data}) P(\text{data}) = P(\text{data} | \Theta) P(\Theta)
\]

just algebra \quad \Rightarrow P(\Theta | \text{data}) = \frac{P(\text{data} | \Theta) P(\Theta)}{P(\text{data})} \propto P(\text{data} | \Theta) P(\Theta)

\Rightarrow pd_{\Theta}(\Theta | \text{data}) \propto pd_{f}(\text{data} | \Theta) pd_{\Theta}(\Theta) \quad L^2 \text{ vector space property}

\begin{align*}
&\quad P(\Theta | \text{data}) = \frac{P(\text{data} | \Theta) P(\Theta)}{P(\text{data})} \propto P(\text{data} | \Theta) P(\Theta) \\
&\quad \Rightarrow pd_{\Theta}(\Theta | \text{data}) \propto pd_{f}(\text{data} | \Theta) pd_{\Theta}(\Theta)
\end{align*}

4th Probability Axiom
just a constant!

• What’s All the Fuss?
• Doesn’t everybody want to make the best RAM decisions possible?
• Doesn’t everybody want to make fast, easy, objective, and completely defendable RAM decisions?
Why Haven’t You Seen this Before?

- MCMC: only been around for about 20 years, used almost exclusively in medicine in EU, So. America, and Australia.
- Methods to derive objective uncertainty models $p_{d_{obj}}(\Theta)$ were invented in 1939, 1970, and 1995 using information theory, not anything from engineering or reliability.
- Without $p_{d_{obj}}(\Theta)$, purely objective decision formulations are impossible, so assumptions are used.
- Without MCMC, you could not solve the objective integrals or do the integral transformations.
- You cannot build a general MCMC tool for all problems.
- Why bother to derive equations you cannot solve?
- Why bother to derive equations in an engineering class that the student will never solve, analytically or numerically?
Real World Examples

- US Coast Guard C130 Cooling Turbine: Selection of the minimum cost Preventative Maintenance interval (IEEE AC 2010, Paper #1238)
- International Space Station EVA O₂ Sensor Drift: Redesign and halt EVA’s for two years, or compensate for Sensor Drift (IEEE AC 2011, Paper #1653)
- F/A-18 Jet Engine reliability as a function of numbers of repairs: Optimize refurbishment intervals (IEEE AC 2012, paper #1498)
- HU-25 AC Generator: Detection and confirmation of multiple failure modes (IEEE AC 2012, paper #1497)
US Coast Guard C130 Cockpit Cooling Turbine

- Cooling Turbine Provides Cooling and Pressurization to the C130 Crew
- Failure in Service
  - Loss of Cooling, but More Important, Loss of Cabin Pressurization
  - Smoke, Loud, Crew Must Secure
  - Mission Compromised
- Costs
  - Replacement: $30,000
  - Refurbishment: $500
The C130 PM Problem

• 60:1 Cost Ratio, Replacement: Refurbish

• Only Had Five Failure Data: 463, 538, 1652, 1673, and 2462 flight hours

• Only Had One Survivor Datum: 96 flight hours

• What PM Interval to select for refurbishment?

• USCG Decision Makers Paralyzed
Step 1

- Validated Weibull Model for $p_{df}(t_f | \eta, \beta)$
- Formed likelihood and got $p_{dobj}(\eta, \beta)$
- Decision discriminator is Cost Savings per Flight Hour – can be written as a function of $\eta$ and $\beta$ and the PM interval to be used

$$CS(t_{pm}) \equiv \left[ \left( \frac{C_{rep}}{\eta} \right)^{\gamma \left( \frac{\beta - 1}{\beta} \right)} \left( \frac{t_{pm}}{\eta} \right)^{\beta} \right] - \left( \frac{C_{pm}}{t_{pm}} \right) \times e^{-\left( \frac{t_{pm}}{\eta} \right)^{\beta}}$$

where $C_{rep}$ is Cost for Replacement,
$t_{pm}$ is the PM Interval for Refurbishment, and
$C_{pm}$ is the Cost for PM

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Step 2

MCMC Joint Samples
Steps 3 and 4

- **Step 3:** Evaluated $CS(t_{pm})$ at $\eta, \beta$ joint samples at $t_{pm}$ from 1 – 3,000 hours
- **Step 4:** Plotted 5th and 95th parameterized quantiles
- Is the *sweet spot* obvious to you?
- USCG picked 250 hours for PM interval
- USCG 95% certain of saving at least $17 per flight hour

**Cost Savings Risk Parameterization**

- 5% Risk of Greater Cost Savings
- Most Likely Cost Savings
- 95% Risk of Greater Cost Savings

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ISS EVA $O_2$ Sensor Drift

- On the International Space Station (ISS), The Extra-Vehicular-Activity (EVA) $O_2$ Sensor Measurements Drifting
  - Sensor Accuracy Requirement: $\pm 6$mmHg for 270 Days post Calibration
  - Errors $> 6$mmHg: Astronaut May Suffer Bends during EVA
  - Errors $< -6$mmHg: Astronaut May Suffer Oxygen Toxicity
  - Either may result in Death of Astronauts
- NASA Faced with Either
  - Halting ISS EVA's Until Sensor Redesign, Testing, and Deployment
  - Or, Compensating for the Error Drift to Reduce the Risk
- Drift Compensation Results were not Convincing
Observed Sensor Errors

- Drift errors for Five Sensors
- All Appeared to Drift in Same Direction, with Similar Rates
- Compensation for Drift Might Reduce the Risk Enough

Compensation Scheme: Use Least Squares on All Data to Estimate Slope and Intercept, and Remove from Sensor Drift Errors

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Sensor Errors After Drift Compensation

- Unacceptable Drift Errors Occur Even Earlier!
- Did the Risk Actually Increase?
- What was the Risk without Drift Compensation?
- No Answers, No Decision!
Step 1

- Validated a covariate model for the drift errors
  \[ \mu_s(TSC) = \mu_0 + \mu' TSC; \ TSC \equiv \text{Time Since Calibration} \]
  \[
pd(e_s | \mu_0, \mu', \sigma_s, TSC) = \frac{1}{\sqrt{2\pi \sigma_s}} e^{-\frac{1}{2} \left( \frac{e_s - \mu_0 - \mu' TSC}{\sigma_s} \right)^2} \]

- Formed likelihood and got \( pd_{obj}(\mu_0, \mu', \sigma_s) \propto 1/\sigma_s \)

- Risk uncertainty model
  \[
pd\left( R\left( |e_s| > e_{max} | \mu_0, \mu', \sigma_s, TSC \right) | \text{data} \right) = \frac{1}{\sigma_s} e^{-\frac{1}{2} \left( \frac{e_{si} - \mu_0 - \mu' TSC_i}{\sigma_s} \right)^2} \times \left( \frac{1}{\sigma_s} \right) \]
  \[ \propto 2 \Phi(-e_{max} | \mu_0 + \mu' TSC, \sigma_s) \times \prod_{i=1}^{N_{es}} \frac{1}{\sigma_s} e^{-\frac{1}{2} \left( \frac{e_{si} - \mu_0 - \mu' TSC_i}{\sigma_s} \right)^2} \times \left( \frac{1}{\sigma_s} \right) \]

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Slide # 41
Steps 3 and 4

- **Step 3**: Evaluated Risk with compensation and without at $\mu_0, \mu', \sigma_s$ joint samples
- **Step 4**: Plotted Risk Density Strips for both
  - Left edge of strip is 5th quantile
  - Right edge is 95th quantile
  - Color density $\propto$ probability density
  - Black bar is at Mode
- NASA chose to go with Drift Compensation
F/A-18 Jet Engine Reliability after Repairs

• The F/A-18 E/F Super Hornet, General Electric F414 Low Bypass Gas Turbine Engine is repaired after failure, and returned to service
• What happens to reliability after repairs?
• How would each successive repair affect mission assurance?
Step 1

- Validated a covariate Weibull model for failures as a function of number of repairs

\[ \eta(N_r) = \eta_0 * e^{\eta_c * N_r} \; ; \; \beta(N_r) = \beta_0 * e^{\beta_c * N_r} \]

\[ pd(t_f \mid N_r, \eta_0, \beta_0, \eta_c, \beta_c) = \frac{\beta_0 * e^{\beta_c * N_r}}{\eta_0 * e^{\eta_c * N_r}} \left( \frac{t_f}{\eta_0 * e^{\eta_c * N_r}} \right) \beta_0 * e^{\beta_c * N_r} - 1 \]

- Developed likelihood for failure and survivor data as function of number of repairs, and got

\[ pd_{obj}(\eta_0, \beta_0, \eta_c, \beta_c) \propto \frac{1}{\eta_0} \left( \frac{1}{\beta_0} \right) \]
Step 2
Steps 3 and 4

- **Step 3**: Evaluated reliability as a function of number of repairs as a function of TSR at $\eta_0$, $\beta_0$, $\eta_c$, $\beta_c$ joint samples
- **Step 4**: Plotted reliability 5$^{th}$ and 95$^{th}$ quantiles as a function of service life and numbers of repairs, and how PM as a function of number of repairs could be determined
The USCG HU-25 Falcon

- The AC Generator was Failing, *Often*
  - 45 Failures
  - 41 Survivors
- But the *data* looked funny
  - Were multiple failure modes in play?
  - If so, finding them could really help focus on the problem parts.
Step 1

- Validated a Weibull mixture model for failures allowing for two failure modes

\[
pd(t_f \mid \gamma, \eta_a, \beta_a, \eta_b, \beta_b)
\]

\[
= \gamma \left( \frac{\beta_a}{\eta_a} \right) \left( \frac{t_f}{\eta_a} \right)^{\beta_a-1} e^{\left( \frac{t_f}{\eta_a} \right)^{\beta_a}} + (1 - \gamma) \left( \frac{\beta_b}{\eta_b} \right) \left( \frac{t_f}{\eta_b} \right)^{\beta_b-1} e^{\left( \frac{t_f}{\eta_b} \right)^{\beta_b}}
\]

- Developed likelihood for failure and survivor data, and got

\[
pd_{obj}(\gamma, \eta_a, \beta_a, \eta_b, \beta_b) \propto (\gamma^{-1/2}) (1 - \gamma)^{-1/2} (1/\eta_a)(1/\beta_a)(1/\eta_b)(1/\beta_b)
\]
In validation, the Markov chains would not stabilize at all if multiple modes did not exist in the data!
Steps 3 and 4 weren’t really necessary.

Previous plots identified an infant mortality failure mode and a wearout failure mode via the data.
Summary

• Slides can be found at www.attwater.com/IEEE 2014 Tutorial.pptx

• There is nothing to Fuss about!

• There is really no reason to call it Bayesian Reliability Analysis.

• The keys:
  • Objective Uncertainty Models for your validated failure uncertainty model
  • MCMC – a new numerical method

• We can now make good, completely objective and defendable, RAM decisions, based solely on what we know for absolute certain, the data
Contact Information

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References


lnpd<-function(tfails,tsurvs,eta,beta) { # log posterior
  return(sum(dweibull(tfails,scale=eta,shape=beta,log=TRUE))
+sum(pweibull(tsurvs,scale=eta,shape=beta,log.p=TRUE,lower.tail=FALSE)
  -log(eta)-log(beta))
}
sampeta<-function(tfails,tsurvs,eta,beta,deta){
  temp<-eta
etastar<-eta+runif(1,-deta,deta)  #get proposed eta
  if (etastar>0) {
    lnalpha<-lnpd(tfails,tsurvs,etastar,beta)-lnpd(tfails,tsurvs,eta,beta)
    lnu<-log(runif(1,0,1)) # get test parameter
    if (lnu<lnalpha) temp<-etastar  # accept and replace
  }
  return(temp)
}
sampbeta<-function(tfails,tsurvs,eta,beta,dbeta){
  temp<-beta
  betastar<-beta+runif(1,-dbeta,dbeta)  #get proposed beta
  if (betastar>0) {
    lnalpha<-lnpd(tfails,tsurvs,etastar,beta)-lnpd(tfails,tsurvs,eta,beta)
    lnu<-log(runif(1,0,1)) # get test parameter
    if (lnu<lnalpha) temp<-betastar  # accept and replace
  }
  return(temp)
}
ws<-function(n=3000,tfails=NULL,tsurvs=NULL,eta0=1000,
beta0=1,deta=60,dbeta=0.5) {
  eta<-eta0
  beta<-beta0
  etas<-eta0
  etaacc<-1
  betas<-beta0
  betaacc<-1
  for (i in 2:n) {
    etanew<-sampeta(tfails,tsurvs,eta,beta,deta)
    etas<-append(etas,etanew)
    if (etanew!=eta) {
      eta<-etanew
      etaacc<-etaacc+1
    }
    betanew<-sampbeta(tfails,tsurvs,eta,beta,dbeta)
    betas<-append(betas,betanew)
    if (betanew!=beta) {
      beta<-betanew
      betaacc<-betaacc+1
    }
  }
  temp<-list(etas=etas,betas=betas)
  cat("Intital Eta: ",eta0,"\n")
cat("  Step: ",deta,"\n")
cat(" ETA acceptance ratio: ",etaacc/n,"\n")
cat("Intital Beta: ",beta0,"\n")
cat("  Step: ",dbeta,"\n")
cat(" BETA acceptance ratio: ",betaacc/n,"\n")
return(temp)
}

In the R programming language

Tutorial: Bayesian Reliability Analysis, What’s All the Fuss? - IEEE AC 2014
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