Quantitative Risk Assessment

Tutorial H02

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Welcome

• Professional Development Tutorial
  • For Practicing Systems Engineers who want to Expand Their Systems Engineering Skills
  • More Specifically, For Systems Engineers who are Working in Risk Management and want to Learn Better Ways to Perform Risk Assessment

• Objectives
  • Review Some Fundamental Concepts that May have been Forgotten
  • Introduce Some Relatively New Improvements for State of the Art Risk Assessment
Topics

• Introduction
• Risk Assessment, and the Risk Management Process
• Qualitative vice Quantitative Risk Assessments
• The Bayesian Approach to Risk Assessment
• Avoiding Unnecessary Assumptions
• Introduction to Markov Chain Monte Carlo Methods
• How to Perform a Quantitative Risk Assessment
• Case Studies
• Summary and Conclusion
• Contact Information and References
Attendee Introductions
(Time Allowing)

- Your Name, Employer, and Type of Business
- Your Job and Type of Work You Do
- Do You Work with Risk or Risk Management?
- What you are hoping to gain from today’s tutorial
Your Tutorial Instructor: Mark A. Powell

- Director, Decision and Risk Technologies at a Fortune 500 Company
- Professor, Systems Engineering, Stevens Institute of Technology, University of Houston
- Over 35 years Experience in Systems Engineering and Project Management
- Past Chair, INCOSE Risk Management Working Group
- INCOSE Technical Leadership Team, Assistant Director for Systems Management Processes
- Contact Information at the End of Tutorial Presentation, Contact Welcomed
Risk Assessment, and the
Risk Management Process

Oft Forgotten Foundations
The Concept of Risk

• What is Risk?
  • Simply An Uncertain Future Consequence
    • The Level of the Consequence is Generally Uncertain
    • The Measure of Risk, or Risk Level:
      The Probability that the Consequence above or below a Specified Level will be Realized
  • In General, the Consequence may be Adverse, or Advantageous
    • If Only Adverse, We may want to Expend Resources to Reduce the Chance of Realization of the Consequences at or Worse than this Level
    • If Only Advantageous, We may want to Expend Resources to Improve the Chance of Realization of the Consequences at or Better than this Level
    • If Both are Possible for a Single Consequence, We may want to Expend Resources to Push Realization of the Consequences from Adverse towards Advantageous
Risk Assessment

- Risk and Risk Assessments are the *Basis for All Decisions*
- Risk Assessment Answers a Simple Question: *Based on the Available Data, How Sure can we be that the Risk Level is Unacceptable?*
- The Decision: Based on the Available Data,
  - If We are *Sufficiently Sure* that the *Probability is too High* that Sufficiently Adverse Consequences will be Realized, *We Decide to Expend Resources to Reduce that Probability*
  - If We are *Sufficiently Sure* that the *Probability is too High* that Sufficiently Advantageous Consequences will NOT be Realized, *We Decide to Expend Resources to Reduce that Probability*
- *Sufficiently Sure, Sufficiently Adverse, Sufficiently Advantageous, and Too High Probability* are all Decision Maker Judgments based on Decision Maker Values for the Specific Decision
The Concept of Managing Risk

• At Heart, A Simple Decision Process
  • Three Alternatives to be Selected Based on Risk Level
    • High – Expend Resources to Reduce Risk Level
    • Moderate – Plan for Reduction and Monitor Risk Level Closely and Frequently over Time
    • Low – Monitor Risk Level Much Less Frequently
  • Must Decide what Combinations of Consequence Levels and Probability Levels Define High, Moderate, and Low

The Familiar n X m Risk Matrix

• Green are Low Risks
• Yellow are Moderate Risks
• Red are Unacceptable Risks
More on the Risk Management Concept

- The Risk Matrix Illustrates the *Decision Structure*
- Risk Assessment Provides the *Decision Discriminator*
  - Statistically Processes Data to Produce an *Assurance* (or Probability) for the Level of Risk
  - The Level of Assurance Required for Action is Determined by the *Project Risk Strategy*
- Both Require *Decision Maker Judgments* based on *Decision Maker Values*
Example: NASA Orbital Debris Avoidance

- A Collision between a Large Piece of Orbital Debris and the Space Shuttle or Space Station would be Catastrophic
- If the Risk of Collision is Too High
  - The Shuttle or Station can Maneuver out of the way of the Debris
  - But, the Maneuver ruins Microgravity Experiments and Causes Expensive Replanning
- NASA’s Risk Based Decision
  - If $P(\text{coll}) > 10^{-4}$, Then Maneuver out of the Way
  - If $10^{-5} < P(\text{coll}) < 10^{-4}$, Then Plan the Maneuver, Don’t Execute, but Monitor $P(\text{coll})$ Frequently
  - If $P(\text{coll}) < 10^{-5}$, Just Monitor $P(\text{coll})$ Infrequently
Possible NASA Strategies

The NASA Risk Decision:

- Debris and Shuttle/Station Tracking Data are Statistically Processed to Produce the Assurance Level for $P(\text{coll}) > 10^{-4}$
- **Strategy 1:** Maximize Vehicle and Crew Protection
  - If Assurance Level (for $P(\text{coll}) > 10^{-4}$) > 10%, Then Maneuver
  - Vehicle and Crew are more Important than Experiments
- **Strategy 2:** Maximize Experiment Protection
  - Don’t Maneuver *Unless* Assurance Level (for $P(\text{coll}) > 10^{-4}$) > 90%
  - Experiment and Replanning Costs Worth Risk to Vehicle and Crew
Risk Management
Processes

• Risk Planning – Establish Procedures for Conducting Risk Management on the Project
• Risk Identification – Discovery of Unanticipated Uncertain Future Consequences during the Project
• Risk Analysis – Establish Root Causes and Sensitivities
• Risk Assessment – Statistically Process Data to Determine Assurance of Risk Level
• Risk Mitigation – Plan and Execute a Project to Reduce or Eliminate Risk Level
• Risk Tracking and Control – Monitor and Measure Risk Management on the Project
• Risk Communication – Explaining How Project Success is Being Assured
Qualitative vice Quantitative Risk Assessments
Qualitative Risk Assessments

• Risk Assessment Can be Purely Qualitative
  • *Mentally* Processing (Mentally doing the Statistics) the Available Data with a lot of Subjectivity thrown in (assumptions) to get an Estimate (SWAG) of Assurance
  • This Assurance Estimate is only as good as the Assessor:
    • *The Assessor’s Brain*: Mental Algorithms and Statistical Processing, Personal Knowledge of the Data
    • The Assumptions that were used, *if even Known*
  • Often Referred to as *Seat of the Pants, Shoot from the Hip, Best Engineering Judgment, Heuristics, or, just a Guess*
  • Sometimes Good Enough, But Usually *Very Dangerous to Use for Serious Risks*
Quantitative Risk Assessments

- Mathematically Models Uncertainty, Consequence Root Causes, and Sensitivities
- Process All Available Data using Statistical Methods to Obtain Quantitative Measures of Assurance of Risk Levels
- Possible Data Types
  - Observed Events or Phenomena
  - Observations that Events have Not Occurred (Censored Data)
  - Data from Similar Problems
  - Heuristics or Professional Opinion
- Proper Quantitative Risk Assessments Use Minimal Assumptions, are Very Reliable and Repeatable
Then, Why Don’t We Always Use Quantitative Risk Assessments?

• Mathematical Formulations (Using Bayesian Decision Theory) for Real World Problems, using All Available Data without Assumptions, almost always become Analytically Intractable
• Even Numerical Approaches (Monte Carlo Methods) Rarely Provide Solutions
• Classical Statistical Methods do not use Censored Data or Heuristics very Well, if at All, Nor Outliers, Which we Do Want to Use
• Classical Statistical Methods use Many (Usually Unstated) Assumptions, Which we Do NOT Want
The State of Risk Assessment Practice Today

• For Real World Risk Assessment Problems
  • Cannot do Proper Quantitative Risk Assessments for Most
  • Because of Futility, Proper Mathematical Formulations Not Taught in most College Engineering Programs
• Current Quantitative Risk Assessment Practices
  • Use of Classical Statistics, if Possible, Usually Ignoring Good Data (Censored and Heuristic) with Many Unstated Assumptions, and Ignoring Outliers
  • Use of Assumptions in Monte Carlo (numerical approximation) Simulations (often called Probabilistic Risk Assessment)
  • Impossible to Perform using only Censored and/or Heuristic Data with Classical Methods
  • Oversimplifications of Problem Formulations to Enable Use of Classical Statistics and/or Assumptions
• Almost Universally, Obtain Overconservative Assurances for Risk Levels with Greatly Increased Mitigation Costs
The State of Risk Assessment in SE Today

• Almost Always Concerned with Risks for Low Probability Events with Severe Consequences
  • Reliability
  • Mission Assurance
  • Safety
• Naturally, Few if any Observed Event Data
• Cannot Use Classical Statistical Methods
  • Many Censored Data, Few Observed Data
  • Cannot Afford Conservatism
• Real World SE Problem Complexity Almost Always Leads to Exclusive Use of Qualitative Risk Assessments
• This is No Longer Necessary!
The Bayesian Approach to Risk Assessment
Bayesian Methods

• All Decision Theory and Analysis uses Bayesian Methods for Risk Assessments
• Very Different from Classical Statistics
  • Produces Full Probability Distributions for Model Parameters, not Just Point Estimates
  • Balances and Fuses All Data Types, and with Prior Uncertainties
  • Allows Direct Computation of Risk Assurance
  • Much Easier than Classical Methods – Only One Simple Recipe to Remember

Very Natural, Like We Human Beings Think!
The Foundation: Bayes’ Law

• Bayes Published in 1763
• Laplace Republished in 1812
• Jeffreys Republished again in 1939
• Analytical Derivation from 4th Probability Axiom
  • \( P(A,B|H) = P(A|B,H)P(B|H) = P(B|A,H)P(A|H) \)
  • Now consider only the Rightmost Equality
    \( P(A|B,H)P(B|H) = P(B|A,H)P(A|H) \)
  • \( P(A|B,H) = P(B|A,H)P(A|H)/P(B|H) \)
  • That’s it!

Notation Conventions:
“,” means Boolean AND
“|” means “Given that”
“H” is our Assumptions
That Bayesian Recipe

• **Bayes’ Law:** \( P(B|A,H) = P(A|B,H)P(B|H)/P(A|H) \)
  • If \( B \) is a Proposition, and \( A \) is Data, we get \( P(B|\text{Data},H) = P(\text{Data}|B,H)P(B|H)/P(\text{Data}|H) \)
  • Now, \( P(\text{Data}|H) \) is just a *Constant Marginal* Probability, and *unimportant*, so we can ignore it and say \( P(B|\text{Data},H) \propto P(\text{Data}|B,H)P(B|H) \)

• The *Interpretation*
  • \( P(B|H) \) is called the *Prior* - the Marginal Probability (Uncertainty) on the Proposition *before* getting the Data
  • \( P(\text{Data}|B,H) \) is called the *Likelihood* - the Probability (Uncertainty) of Getting the Data Given the Proposition is True
  • \( P(B|\text{Data},H) \) is called the *Posterior* - the Probability (Uncertainty) on the Proposition *after* the Probability of Getting the Data Given the Proposition is Compounded with the *Prior*

**The Bayesian Mantra:**

*Posterior \( \propto \) Likelihood \( \times \) Prior*
A Convenient Property of Bayes’ Law

- Bayes’ Law as Derived and Presented Works with Probabilities
  \[ P(B \mid Data, H) \propto P(Data \mid B, H) P(B \mid H) \]

- Probabilities are Always Integrals of Probability Density Functions
  \[ P(x \leq X \mid H) = \int_{x_{min}}^{X} p_d(\xi \mid H) d\xi \]

- Bayes’ Law also Works for Probability Density Functions
  \[ p_d(B \mid Data, H) \propto p_d(Data \mid B, H) p_d(B \mid H) \]

- And, Works for Combinations of Probability Density Functions and Probabilities
  \[ p_d(B \mid Data, H) \propto P(Data \mid B, H) p_d(B \mid H) \]

This is Very Useful and Convenient for Decision Theory/Analysis and Risk Assessment
The Posterior

• Remember the Bayesian Mantra
  \[ \text{Posterior} \propto \text{Likelihood} \times \text{Prior} \]

• Just *Multiply* the Likelihood Formula Times the Prior Formula

• Produces the *Probability Model* for the Parameter Values for our Uncertainty Model for our Decision Discriminator

• Sounds Complicated, but Really is *Simple*
The Likelihood

• We Select a Probability Model to Represent our Uncertainty about the Consequence
• We are Uncertain about the Specific Shape of the Model, i.e., Uncertain about the Values of the Probability Model Parameters
• The Likelihood is then the Probability of getting our Data given our Model and Hypothetical Values of its Parameters
  • The Likelihood is a Probability Statement that Fuses All our Data, Observed Event Data and Other Information
  • For Event Data, the Product of the Probability Density Function evaluated at the Event Data - the same Likelihood Function used in Maximum Likelihood Estimation
  • For Other Information, a Simple Probability Statement
  • We can write and code these Mathematically
• Most Often, Our Data are Independent – The Likelihood is then just a Product of Probabilities or Densities, or a Product of Both
A Likelihood Example

• Decision: Is our Product Reliable Enough at 250 Hours?
  • Selected Model: Weibull
    $$p_d(t_f \mid \eta, \beta) = \left( \frac{\beta}{\eta} \right) \left( \frac{t_f}{\eta} \right)^{\beta - 1} e^{-\left( \frac{t_f}{\eta} \right)^{\beta}}$$
  • Data:
    • Failures at 463, 538, 1652, 1673, and 2562 hours
    • Survivor at 96 hours (censored datum)
  
  $$P(t_f \geq T \mid \eta, \beta) = e^{-\left( \frac{T}{\eta} \right)^{\beta}}$$

• We Simply Write Out the Likelihood
The Likelihood Formula

- Failure Terms are Products of Weibull Density Evaluated at each Failure Time

\[ l_f(463, 538, 1652, 1673, 2462 \mid \eta, \beta) = \left( \frac{\beta}{\eta} \right)^5 \left( \frac{463}{\eta} \right)^{\beta-1} e^{-\left(\frac{463}{\eta}\right)^\beta} \times \left( \frac{538}{\eta} \right)^{\beta-1} e^{-\left(\frac{538}{\eta}\right)^\beta} \times \left( \frac{1652}{\eta} \right)^{\beta-1} e^{-\left(\frac{1652}{\eta}\right)^\beta} \times \left( \frac{1673}{\eta} \right)^{\beta-1} e^{-\left(\frac{1673}{\eta}\right)^\beta} \times \left( \frac{2462}{\eta} \right)^{\beta-1} e^{-\left(\frac{2462}{\eta}\right)^\beta} \]

- Survivor Term is Probability that Failure would Occur after Survivor Time, \( P(t_f > 96 \mid \text{Weibull, } \eta, \beta) \), or the Reliability

\[ l_s(96 \mid \eta, \beta) = e^{-\left(\frac{96}{\eta}\right)^\beta} \]

- Overall Likelihood – The Product of it All

\[ l(t_f = (463, 538, 1652, 1673, 2462), t_s = 96 \mid \eta, \beta) = \left( \frac{\beta}{\eta} \right)^5 \left( \frac{463}{\eta} \right)^{\beta-1} e^{-\left(\frac{463}{\eta}\right)^\beta} \times \left( \frac{538}{\eta} \right)^{\beta-1} e^{-\left(\frac{538}{\eta}\right)^\beta} \times \left( \frac{1652}{\eta} \right)^{\beta-1} e^{-\left(\frac{1652}{\eta}\right)^\beta} \times \left( \frac{1673}{\eta} \right)^{\beta-1} e^{-\left(\frac{1673}{\eta}\right)^\beta} \times \left( \frac{2462}{\eta} \right)^{\beta-1} e^{-\left(\frac{2462}{\eta}\right)^\beta} \times e^{-\left(\frac{96}{\eta}\right)^\beta} \]
The Prior

- We Select Models for our Uncertainty about the **Values of the Parameters** of the Probability Model we Selected for the Consequence
- This can be Quite Challenging
  - We Usually have no way to tell What an **Appropriate Model** would be for the Values of the Parameters for Some Mathematically Complex Probability Model for the Consequence
  - We are **Effectively Ignorant** about the Models to be Selected for the Prior
- This is Where **Assumptions** Usually are Made
  - Prior Models are just Assumed, that when Multiplied by the Likelihood, produce a Nice Recognizable Posterior Model that is Analytically Tractable and Integrable
  - These Assumptions for the Prior without any Physical Meaning can be **Very Dangerous** in Risk Assessment
- There is an **Easy Solution** to Avoid Making These Unnecessary Assumptions
Avoiding Unnecessary Assumptions
Priors Problems

- Sometimes We Cannot or Do not Want to Specify a Model for our Prior Uncertainty
- We model our Prior as if We are Ignorant! We Want to be as Objective as Possible!
  - New Projects - Never been Done Before
  - Lack of Heuristics
  - Lack of Engineering Judgment
  - Misguided Desire to be Objective
  - Need to Set a Reference Baseline
  - Need to Set a Worst Case Scenario
- But how do you Model Ignorance or Objectivity about Uncertainty?
Specifying Uncertain Objectivity

- There are Three Ways to Model Objectivity for Uncertainty for any Particular Problem
  - Find the Prior Model that Minimizes the Fisher Information Matrix - Jeffreys’ Priors
  - Find the Prior Model that Maximizes the Entropy - Maximum Entropy Priors
  - Find the Prior Model that Maximizes the Expected Value of Perfect Information - Reference Priors
- Most Often Objective Priors are Improper
  - Usually Cause Posterior to be Improper
  - Integral is Infinite or Undefined
  - Not a Cause for Concern except for Analytical Solutions, and then often Not a Concern
General Objective Priors

- All Three Derivation Methods Produce the Same Objective Prior Models for the Same Problem (Consequence Model)
- *This is Extremely Reassuring!*
- *No Need to Derive* Objective Priors Models Using these Methods – Already Documented in Textbooks
- Based on Information Theory, Objective Priors Provide the Absolute Most Objectivity Possible
Notes on Objective Priors

- **Objective Priors** are often called **Non-Informative Priors**
- **Jeffreys’ Priors** for Scale and Shape Parameters Generally Produce **Improper** Posteriors
  - Not Integrable Analytically
  - Must Resort to *Monte Carlo* Integration Approaches
- In some Cases, Objective Priors (even when Improper) are also **Conjugate Priors** (they Produce **Proper** Posteriors!)
  - Flat *(Jeffreys)* for Binomial Likelihood - Beta Posterior
  - Flat *(Jeffreys, Improper)* for Normal *(σ² known)* Likelihood - Normal Posterior *(Proper)*
  - Inverse *(Jeffreys, Improper)* for Gamma Likelihood - Gamma Posterior *(Proper)*
  - Inverse *(Jeffreys, Improper)* for Normal *(μ known)* - Gamma *(or χ²)* Posterior *(Proper)*
- **Improper Objective Priors** that are also **Conjugate** are **Amazing!**
Using Objective Priors Practically

- Look at Likelihood Model $p_d(data|parameters,H)$
  - Identify Type of Parameters for that Model that need Priors (Location, Scale, or Shape)
  - Use an Objective Prior for each Parameter according to its Type (Look it up in One of the References)
  - Assume *Independence* between Parameters
    $p_d(p_1,p_2,\ldots,p_n|H) = p_d(p_1|H)p_d(p_2|H)\ldots p_d(p_n|H)$
  - Because of Independence, Our Joint Prior is simply the product of these Objective Priors
- Multiply our Joint Objective Prior times our Likelihood to get our Posterior
  $p_d(p_1,p_2,\ldots,p_n|data,H) = p_d(data|p_1,p_2,\ldots,p_n,H)p_d(p_1|H)p_d(p_2|H)\ldots p_d(p_n|H)$
Objective Priors for Common Models

- Usually Simple Models
  - For all Location Parameters (e.g., \( \mu \) for the Gaussian), Use 1
  - For the Exponential Family (includes Gaussian, Exponential, Gamma, Weibull, etc.) of Models
    - Scale Parameters (e.g., \( \sigma \) for the Gaussian), Use an Inverse (e.g., \( 1/\sigma \) for the Gaussian, \( 1/\eta \) for the Weibull)
    - Shape Parameters (e.g., \( \beta \) for the Weibull), Use an Inverse (e.g., \( 1/\beta \) for the Weibull)
  - For Discrete Bernoulli Based Models (includes Binomial, Negative Binomial, Geometric, etc.), Use \( \theta^{-1/2}(1-\theta)^{-1/2} \)
  - For Poisson Model, Use \( \lambda^{-1/2} \)

- Bernardo and Smith Has Reference Priors for Other Models
Using Objective Priors Practically, an Example

• Example: Normal Likelihood, Want to use Objective Priors for Uncertain Mean and $\sigma$

  • We want the Posterior:
    \[
    p_d(\mu, \sigma \mid x_1, x_2, \ldots, x_n, H) \propto p_d(x_1, x_2, \ldots, x_n \mid \mu, \sigma, H) p_d(\mu, \sigma \mid H)
    \]

  • Our Likelihood:
    \[
    p_d(x_1, x_2, \ldots, x_n \mid \mu, \sigma, H) \propto \sqrt{\frac{1}{\sigma^2}} e^{-\left(\frac{1}{2}\right)\sum_{i=1}^{n} \frac{(x_i - \mu)^2}{\sigma^2}}
    \]

  • Mean is Location Type - Use Flat Prior $p_d(\mu \mid H) \propto 1$
  • $\sigma$ is Scale Type - Use Inverse Prior $p_d(\sigma \mid H) \propto \frac{1}{\sigma}$

  • Our Joint Objective Prior for this Problem
    \[
    p_d(\mu, \sigma \mid H) = p_d(\mu \mid H) p_d(\sigma \mid H) \propto 1 \times \frac{1}{\sigma} = \frac{1}{\sigma}
    \]

  • Our Joint Posterior:
    \[
    p_d(\mu, \sigma \mid x_1, x_2, \ldots, x_n, H) \propto \sqrt{\frac{1}{\sigma^2}} e^{-\left(\frac{1}{2}\right)\sum_{i=1}^{n} \frac{(x_i - \mu)^2}{\sigma^2}} \times \frac{1}{\sigma}
    \]
Some Notes on our Example

- Our Priors were Improper
  \[ \int_{-\infty}^{\infty} p(\mu | H) d\mu = \int_{-\infty}^{\infty} 1 d\mu = \infty ; \int_{0}^{\infty} p(\sigma | H) d\sigma = \int_{0}^{\infty} \frac{1}{\sigma} d\sigma = \infty \]

- Our Posterior is Pretty Ugly
  \[ p_d(\mu, \sigma | x_1, x_2, \ldots, x_n, H) \propto \frac{1}{\sigma^2} e^{-\frac{1}{2} \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^2}} \]
  - Obviously a Joint Posterior
  - An Improper Joint Posterior
  - Analytical Integrals are Not Possible
  - Numerical Integral Approximations Are Only Possible
    with Markov Chain Monte Carlo Methods
Posterior for Our Earlier Likelihood Example

• Our Reference Priors for the Weibull Model are $1/\beta$ and $1/\eta$

• We Simply Write Out Our Posterior

\[ \text{Posterior} \propto \text{Likelihood} \times \text{Priors} \]

\[ p(\eta, \beta \mid t_f = (463, 538, 1652, 1673, 2462), t_s = 96) \propto \left( \frac{\beta}{\eta} \right)^5 \left( \frac{463}{\eta} \right)^{\beta - 1} e^{-\left(\frac{463}{\eta}\right)} \times \left( \frac{538}{\eta} \right)^{\beta - 1} e^{-\left(\frac{538}{\eta}\right)} \]

\[ \times \left( \frac{1652}{\eta} \right)^{\beta - 1} e^{-\left(\frac{1652}{\eta}\right)} \times \left( \frac{1673}{\eta} \right)^{\beta - 1} e^{-\left(\frac{1673}{\eta}\right)} \times \left( \frac{2462}{\eta} \right)^{\beta - 1} e^{-\left(\frac{2462}{\eta}\right)} \times e^{-\left(\frac{96}{\eta}\right)} \times \left( \frac{1}{\eta} \right)^{\beta} \]

• Now, the Sticky Part

  • We Want the Uncertainty for the Reliability
  • Must Transform the Reliability Equation by Our Posterior Uncertainty for $\eta$ and $\beta$
  • *Impossible to Do Analytically – Must Use Markov Chain Monte Carlo Methods*
Pseudo-Ignorance Priors

- Sometimes, *Objective Priors* for Scale or Shape parameters (inverses) produce Posteriors that *do not do well* in *Markov Chain Monte Carlo Integration*
  - The Markov Chain never really *Stabilizes*
  - Create a *Pseudo-Ignorance Prior* - Use some Engineering Judgment to *Truncate* the Inverse at an appropriate Point
  - Improper Ignorance Priors *become Proper when Truncated*
- *Use of Pseudo-Ignorance Priors* should be *Very Realistic* and *Very Reasonable* if Needed
Pseudo-Ignorance Prior Example

- Example: Vehicle Reliability Test using Weibull Model
  - Our Concern is that the vehicle will not meet the Reliability Requirement of 95% at 100,000 miles
  - Key: Our Statistical Inference about Parameter $\eta$
    - $\eta > 269,145$ miles indicates Reliability Requirement Satisfied
    - Jeffrey’s Prior used for scale parameter $\eta (1/\eta)$, Improper
    - Markov Chain not Suitable for Monte Carlo Integration - $\eta$ sampling became Very Unstable
    - Truncated Jeffreys’ Prior for $\eta$ (scale, $1/\eta$) at $\eta = 1,000,000$ miles
    - Reasonable to Do: If Requirement is met, Reliability at 1,000,000 miles is $10^{-23}$
    - Markov Chain Stabilized allowing Monte Carlo Integration
Introduction to Markov Chain Monte Carlo Methods
Inherent Difficulties Using Bayesian Methods

- Ever Wonder Why You were not Trained to Use Bayesian Methods? For Real World Decision Problems and Risk Assessments:
  - The *Posterior* is Rarely a Recognizable Probability Model that we can Integrate or Use
  - The *Posterior* may be *Improper*
  - The *Posterior* may not be *Integrable*, Impossible to Compute Probabilities for Actions or Utilities
  - The *Posterior* may have a very large number of dimensions
  - Applying the *Posterior* Uncertainty to the Decision Discriminator is Usually Impossible
- These difficulties limited use of *Bayesian Methods* to but a few very simple decision applications – until very recently!
- *Real World SE and Decision Problems Require Markov Chain Monte Carlo Methods* to Solve
A New Numerical Method

• Not Really an Advancement in Quantitative Risk Assessment
• New Numerical Methods Allow Proper Quantitative Risk Assessment to be Done
  • In the Mid 1990’s, European Biomedicine Began Using New Markov Chain Monte Carlo Methods to Produce Quantitative Risk Assessments
  • Markov Chain Monte Carlo Methods provide very good numerical approximations for Real World Analytically Intractable Risk Assessment Formulations
    • No Assumptions Necessary, Models of Objectivity Can be Used Instead
    • All Data Types can be Fused into the Assessment and Used Effectively, including Censored and Heuristic
    • No Need to Ignore Outlier Data
Markov Chain Monte Carlo

- A More General Version of Monte Carlo Methods
  - Does Not Require Defined Sampling Models
  - Does Not Require Assumptions, Completely Objective
  - Will Work with Analytically Intractable Formulations
  - Can work for Very High Dimensional Problems (up to 20,000 related sources of uncertainty)
  - Simple Algorithms to Code, but Not Amenable to Packaging as a Computational Tool

- When Used in Risk Assessment, Provides Full Quantitative Assurance of Risk Levels for the Most Complicated Real World Risk Assessment Problems
MCMC Concepts

- Ordinary Monte Carlo Integration Widely Used
  - Used Specifically to Numerically Approximate Probability Integrals
  - Limited to only *Known* Models for Uncertainty
  - Produces Independent samples
  - For Random Variable $X$, to compute $P(X<3)$, with $n$ MC samples of $X$, merely count the number of $X_i<3$ and divide by $n$

- Markov Chain Monte Carlo also produces random samples, but with sample to sample correlation (in a first order Markov Chain)

- For calculating probability Integrals, sample to sample correlation is irrelevant if the Model is adequately sampled
MCMC Advantages

- MCMC allows reasonably complete sampling of any *Posterior* you can Formulate
  - Unknown and Unknowable *Posteriors*
  - Non-Integrable *Posteriors*
  - Improper *Posteriors*
  - High dimension *Posteriors*
- Relatively simple and fast
- Software Packages are available for some Special Problems
- Easy to code up for Unique Problems
- Intuitively Tunable
The Metropolis-Hastings Algorithm

• To Start, formulate the Posterior density $pd(\Theta|data)$, and select a proposal step size $d\Theta$
• Start with any legal value: $\Theta_i = \Theta_1$
• Repeat Loop to get new samples
  • Propose a new sample: $\Theta_{i+1} = \Theta_i + \Delta\Theta$
    where $\Delta\Theta \sim U(-d\Theta,d\Theta)$
  • Calculate the ratio of Posterior densities:
    $\alpha = \frac{pd(\Theta_{i+1}|data)}{pd(\Theta_i|data)}$
  • Obtain a sample $u$ from a uniform distribution: $u \sim U(0,1)$
  • If $u < \alpha$, then accept the proposed sample as $\Theta_{i+1}$,
    else set the new sample to the previous one: $\Theta_{i+1} = \Theta_i$
M-H Algorithm Notes

• $\Theta$ can be a vector of parameters $\Theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_n)$
  - Can propose an entire new vector then test for acceptance of the new vector ($d\Theta$ and $\Delta\Theta$ are vectors), or
  - Can add an internal loop to propose each new parameter and test it for acceptance to get a new sample vector

• Each sample of the vector $\Theta$ is a sample of the joint Posterior Model

• Can use other Models for proposal besides $U(-d\Theta,d\Theta)$

• Scatterplots of $\theta_j$ vs $\theta_k$, $j \neq k$, show correlation or marginal joint densities

• Set of $\theta_i$ samples provide Marginal Posterior distributions for the $i^{th}$ parameter:

$$pd(\theta_i \mid data) = \int_{\theta_1} \int_{\theta_2} \cdots \int_{\theta_{i-1}} \int_{\theta_{i+1}} \cdots \int_{\theta_n} pd(\Theta \mid data) d\theta_1 d\theta_2 \cdots d\theta_{i-1} d\theta_{i+1} \cdots d\theta_n$$
Tuning the M-H Algorithm

- Subjective judgment is usually all that is needed
- Need to collect data on numbers of proposed samples accepted (vector or on each parameter)
- Acceptance Ratios should be within 30-60% range, if not, adjust the proposal step size
  - If ratio is too high, samples will show tracking, Increase step size
  - If ratio is too low, samples will get stuck, Decrease step size
- Must look at Burn-In, Convergence to Stationary Markov Chain
MCMC Tuning by Eye

- **Tracking**
  - Acceptance Ratio – 0.9
  - Increase Step Size

- **Stuck**
  - Acceptance Ratio – 0.1
  - Decrease Step Size

- **About Right**
  - Acceptance Ratio – 0.53
  - Suitable Step Size
M-H Convergence to Stationarity

- Scatterplot samples to identify point at which Markov Chain becomes stationary, use samples beyond this point – Called *Burn-In*

Converges before 1,000 samples

Rightmost 9,000 samples
Some M-H Heuristics

• Use short test chains (~1K-3K points) to Tune
  • Means are good starting points for quicker *Burn-in*
  • Two Std. Deviations are good first proposal step sizes

• Parameter-by-parameter (inner loop) acceptance testing is easier to *tune* than vector-at-once acceptance testing

• Beware
  • Changes on proposal step size for one parameter may change acceptance ratios for other parameters
  • *Improper Posteriors* based on censored data can show stationarity in short chains, then track off in longer chains, then back – Need Carefully Constructed *Pseudo-Ignorance Priors*
MCMC Extensions

• Predictions of Distributions of Future Observations – You Can Predict the Future
  • Simply evaluate a Hypothetical Likelihood for a Single Future Datum at all joint samples of $\Theta$ and Average at Points of Interest
  • Then, calculate whatever statistics you need from this distribution

• Uncertainty Models for Functions of the Posterior Uncertainty, or Transforming by the Posterior
  • Easier Process – evaluate function of posterior random variables at all samples (or sample sets)
  • This is What you do with Decision Discriminators
For More Information on MCMC

• Numerous Texts, Check Amazon

• Contact Welcome
  • Articles
  • Examples

• Stevens Course
  • SYS 601: *Probability and Statistics for Systems Engineers*
  • Webcampus: [http://webcampus.stevens.edu](http://webcampus.stevens.edu)
How to Perform a Quantitative Risk Assessment
The Process

- Analyze the Problem and Understand the Consequences
- Identify an Appropriate Uncertainty Model
- Identify all Relevant Data and Information
- Formulate the Posterior
  - Write out the Likelihood Formula for the Data and Information
  - Write out Select Reference Priors Formula
  - Multiply the Likelihood Formula by the Priors Formula
- Write the Code to do a MCMC Sampling of this Posterior
- Tune the Samples by Trial and Error
- Evaluate the Risk or Discriminator at the Samples
- Compute the Needed Probabilities and Decide based on this Risk Assessment
A Walkthrough for a Real World Example

• The NASA Orion Crew Exploration Vehicle Launch Abort System (LAS) must Fire for 23 seconds with Very High Reliability (0.9973)

• We can fire Several LAS Rockets and note Failure and Survivor Times

• We Need to know How Sure We can be That the LAS Reliability exceeds 0.9973

• If We are 90%+ Sure, We Accept the LAS
Step 1: Understand The Thing About Which you are Uncertain

• First: Identify the Uncertainty Question that Needs Answering
  Based on our Data, How Sure Can We be that the Reliability of the LAS SRM’s is greater than 0.9973 for a 23 second firing?

• Second: What is the Specific Item that is Uncertain?
  The LAS Reliability at 23 seconds.

• Third: What are the Data or Information Available or to be Obtained?
  Failure and Survivor Times

• Fourth: What are the Physics or other Related Conditions or Phenomena Associated with the Data and Item?
  Failures can Only Occur After t=0, but May Never Occur at All;
  Failure and Survivor Data are All Independent
Step 2: Identify Suitable Models for the Uncertainty

- Are the Uncertain Items or Data Continuous or Discrete?
  *Failure and Survivor Times are Essentially Continuous*

- Are the Uncertain Items or Data Two-sided, One-sided, or Limited to a Specific Range?
  *Failure and Survivor Times are One-sided*

- Should the Model be Uni-modal, or Multi-modal?
  *There Should be a Single Mode, a Single Most Likely Failure Time – A Weibull Model would work Well*
Step 3: Write Out the Probability Equation

- Based on the Model you Selected, Simply Write Out the Risk Assessment Probability Equation

\[
P \left( R \left( 23 \mid \eta, \beta \right) \geq 0.9973 \mid Data \right) = P \left( e^{-\left( \frac{23}{\eta} \right)^\beta} \geq 0.9973 \mid Data \right)
\]

Now, We Need to Infer from the Data the Uncertainty Model for the parameters of the Weibull Model, \( \eta \) and \( \beta \).
Step 4: Write Out the Likelihood for the Data

- For \( n \) Independent Failure Times \( (f_i) \), the Likelihood is the Product of the Weibull Density Evaluated at these Failure Times

\[
L(f_1, \ldots, f_n \mid \eta, \beta) = \prod_{i=1}^{n} \left( \frac{\beta}{\eta} \right) \left( \frac{f_i}{\eta} \right)^{(\beta-1)} e^{-\left(\frac{f_i}{\eta}\right)^\beta}
\]

- For \( m \) Independent Survivor Times \( (s_i) \) at 23 seconds, the Likelihood is the Product of the Probabilities that the Failure would Occur After 23 Seconds

\[
L(s_1, \ldots, s_m \mid \eta, \beta) = \prod_{i=1}^{m} e^{-\left(\frac{s_i}{\eta}\right)^\beta} = \prod_{i=1}^{m} e^{-\left(\frac{23}{\eta}\right)^\beta}
\]

- Write Out the Total Likelihood

\[
L(f_1, \ldots, f_n, s_1, \ldots, s_m \mid \eta, \beta) = \prod_{i=1}^{n} \left( \frac{\beta}{\eta} \right) \left( \frac{f_i}{\eta} \right)^{(\beta-1)} e^{-\left(\frac{f_i}{\eta}\right)^\beta} \ast \prod_{i=1}^{m} e^{-\left(\frac{23}{\eta}\right)^\beta}
\]
Step 5: Select Prior Models for the Parameters of the Uncertainty Models

- Usually, Objective Priors

For the Weibull Model Parameters:

\[ p(\eta, \beta) \propto \left( \frac{1}{\eta} \right) \left( \frac{1}{\beta} \right) \]
Step 6: Write Out the Formula for the Posterior Model

• Simply Multiply the Likelihood Formula times the Prior Formula

\[
p(\eta, \beta \mid f_1, \ldots, f_n, s_1, \ldots, s_m) \propto L(f_1, \ldots, f_n, s_1, \ldots, s_m \mid \eta, \beta) \times p(\eta, \beta)
\]

\[
= \prod_{i=1}^{n} \left( \frac{\beta}{\eta} \right) \left( \frac{f_i}{\eta} \right)^{(\beta-1)} e^{-\left(\frac{f_i}{\eta}\right)^\beta} \times \prod_{i=1}^{m} e^{-\left(\frac{23}{\eta}\right)^\beta} \times \left(\frac{1}{\eta}\right) \left(\frac{1}{\beta}\right)
\]
Step 7: Create a MCMC Sampler for This Posterior and Obtain a Tuned Set of Samples

- Write out the MCMC Code for this Posterior Joint Density
- Run it Until you Obtain a Good Set of Samples of the Parameters without Burn-in and with a Suitable Acceptance Ratio for Each Parameter
- Obtain a Large Number (10K-100K) of Samples of $\eta$ and $\beta$
Step 8: Calculate the Item of Interest Using Each Set of MCMC Samples

• Create Samples of the Item of Interest by Evaluating at the Samples of the Parameters of the Model

\[ R_i(23) = R(23 | \eta_i, \beta_i) = e^{-\left(\frac{23}{\eta_i}\right)^{\beta_i}} \]
Step 9: Use MC to Answer the Probability Question

- Count the Number of Samples of the Item of Interest Above the Threshold, and Divide by Number of Samples
  \[ \text{Count the Number of } R_i \geq 0.9973 \text{ and Divide by } n \]

- If the Result is above the Probability Threshold, then Act
  *If Result is > 0.9, Accept the LAS*
Case Study 1: Space Shuttle Cargo Transfer Bag Risk Assessment
Space Shuttle Cargo Transfer Bag Test

- Cargo Transfer Bags (CTB) to be Carried on Shuttle to Space Station
- Required Zipper Cycle Life – 2,000 Cycles
- If CTB Zipper Fails During Launch or Descent, Loose Object could Penetrate the Hull
- Performed a Single Test
  - One CTB Only
  - 8,000 Successful Zipper Cycles
- Relevant Question
  
  How Sure can we be from the Test Result that the TRUE Risk of CTB Zipper failure by 2,000 Cycles is below some Acceptable Level?
CTB Quantitative Risk Assessment Setup

- No Clear Decision Rule
  - Decision Discriminator: Risk of Zipper Failure by 2000 Cycles (Planned Lifetime)
  - No Set Acceptable Risk for Zipper Failure for Using CTB’s
  - No Set Acceptable Assurance for Acceptable Risk
- Use Weibull Model for Risk of Failure – one sided, general
- Datum is *one* Test Result: 8,000 Cycles without a Failure
- Posterior: Likelihood $\times$ Prior
  - Priors: Pseudo-Ignorance for $\eta$ ($1/\eta$), $\eta < 30,000$ Cycles;
    Pseudo-Ignorance for $\beta$, ($1/\beta$), $1 < \beta < 20$
  - Likelihood is Probability that Zipper would Not Fail in 8,000 Cycles – Just the Reliability at 8,000 cycles
Quantitative Risk Assessment Results for the CTB Test

- **Test Datum:**
  Successful 8K Cycles without a Failure on One CTB Zipper

- **Assumptions:**
  - Zipper Cycling Cannot Improve Reliability of the CTB Zipper (pseudo-ignorance prior for $\beta$)
  - CTB’s will not be used more than 30,000 Cycles (pseudo-ignorance prior for $\eta$)

- **No Stated Minimum Acceptable Risk – So Parameterize**

<table>
<thead>
<tr>
<th>Risk of CTB Zipper Failure by 2K Cycles ($R_{2K}$)</th>
<th>Assurance Provided by Test Results $P(\text{True } R_{2K} &lt; R_{2K})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>75%</td>
</tr>
<tr>
<td>5%</td>
<td>88%</td>
</tr>
<tr>
<td>10%</td>
<td>94%</td>
</tr>
<tr>
<td>20%</td>
<td>98%</td>
</tr>
</tbody>
</table>
Case Study 2: International Space Station EVA Oxygen Sensor Drift Risk Assessment
ISS O2 Sensor Drift

- Problem: Space Station EVA Oxygen Sensor Measurement Accuracy is Observed to drift with Time
  - If the Measured $O_2$ is in Error by more than $\pm 6$mmHG within 270 days since Calibration, it could Kill an Astronaut
  - Already Compensating for Pressure Variations in Measurement Accuracy (Successful)
- Proposed Solution Options:
  - Test for Drift rates and Compensate for Drift; OR,
  - Redesign $O_2$ Sensor and Ship Up to ISS
- Questions:
  - What is the Existing Risk of Sensor Accuracy Drift Beyond Acceptable Limits?
  - What is the Risk After the Proposed Drift Compensation?
O2 Sensor Test Data

Drift of the CSA-O2s During Long Life Evaluation
(Data is pressure corrected)

Accuracy (mmHg)

Days Since Calibration

270 Days
Drift Corrected O2 Sensor Data

Drift Time-Corrected CSA-O2s During Long Life Evaluation
(Data is pressure corrected)

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ISS O2 Sensor Quantitative Risk Assessment Setup

- No Clear Decision Rule
  - Decision Discriminator: Risk of Measurement Errors Getting too Big by 270 Days since Calibration (Planned Lifetime)
  - No Set Acceptable Risk for Unacceptable Measurement Error
  - No Set Acceptable Assurance for Acceptable Risk
- Use Gaussian Model for Measurement Errors
  - Mean of Errors Appears to Drift with Time
  - Model This: $\mu = \mu_0 + \mu_\text{dot} \cdot t$
  - We are Now Uncertain about $\mu_0$, $\mu_\text{dot}$, and $\sigma^2$
  - errors $\sim N(\mu_0 + \mu_\text{dot} \cdot t, \sigma^2)$
- Data:
  - Pre-drift corrected Measurement Errors (at Times) to Assess Risk Before Drift Correction
  - Post-drift corrected Measurement Errors (at Times) to Assess Risk After Drift Correction
More Setup

- **Posterior: Likelihood \times Prior**
  - **Priors**
    - Reference Priors for $\mu_0$ and $\mu_{\dot{}}$, Just a Simple Constant 1
    - Reference Prior for $\sigma$, the inverse ($1/\sigma$)
  - **Likelihood: Product of Densities at Measurements at Times**
    \[
    \log \text{likes}(\text{err}(t) \mid \mu_0, \mu_{\dot{}} , \sigma) = \log \left( \prod_{i=1}^{n} \left( \frac{1}{\sqrt{2\sigma^2}} \right) \ast e^{-\frac{(\text{err}_i - \mu_0 - \mu_{\dot{}} * t_i)^2}{2\sigma^2}} \right)
    \]
    \[
    = \log \left( \frac{1}{\sqrt{2\sigma^2}} \right) \prod_{i=1}^{n} e^{-\frac{(\text{err}_i - \mu_0 - \mu_{\dot{}} * t_i)^2}{2\sigma^2}} = -n \ast \log(\sqrt{2\sigma^2}) - \sum_{i=1}^{n} \frac{(\text{err}_i - \mu_0 - \mu_{\dot{}} * t_i)^2}{2\sigma^2}
    \]
    \[
    \propto -\frac{n}{2} \ast \log(\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (\text{err}_i - \mu_0 - \mu_{\dot{}} * t_i)^2
    \]
  - **Code: Relatively Straightforward**
Before and After Drift Correction Risk Results

CSA O2 Sensor Accuracy Limit Risk at 270 Days

Linear Scale

Logarithmic Scale

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O2 Sensor PRA Summary

• Without Drift Compensation, Risk of Exceeding Accuracy Limits at 270 Days is 36-46% (with 90% Certainty)

• With Drift Compensation, 95% Sure Risk of Exceeding Accuracy Limits at 270 Days is < 1.5%

• Additional O2 Level Compensation could Reduce Risk Further
Case Study 3: Astronaut Spaceflight Bone Fracture Risk Assessment
Astronaut Spaceflight Fracture Risk

- Need Quantification of Risk of Bone Fracture during Space Missions
- Mission Duration can Vary in Length
- Based on Historical Human Spaceflight Data
- Will Calculate Risk Parameterized as Function of Mission Duration
  - Risk Estimates are Risk Experienced by Past Crewmembers
  - Good Predictor of Risk for Future Missions with No Significant Changes
    - Similar Mission Activities
    - Same Crew Selection Processes
Quantitative Risk Assessment

- Avoid *Hard to Defend* Assumptions
  - Use Objective Priors
  - Model Priors Intelligently (Pseudo-Ignorance)
    - Shape Parameter: $1 < \beta < 20$
    - Scale Parameter: $\eta < 10$ years of $\mu G$ Exposure
- Sample Joint Posterior of $\eta$ and $\beta$ using MCMC
- Calculate Risk of Fracture for Various Assurance Levels using MC Samples of $\eta$ and $\beta$ and Plot
Human Spaceflight
Bone Fracture Data

- No Bone Fractures Reported for any Human Spaceflight Mission
- 977 μG Exposures
  - No Significance to Index # or Order of Data
  - All Crewmembers Included
  - 294 Flights
  - Includes all Russian flights
  - Includes all U.S. flights
  - 1 Chinese Flight
  - 3 Spaceship One flights
  - All ISS Missions as of May 2005
- 56 MIR missions
- Source is Astronautix.com
Astronaut Spaceflight Fracture Risk

- Picked Five Assurance Levels
  - 5, 25, 50, 75, and 95% Levels
  - Black Line is *Expected Risk*
- Ordinate: Risk of Bone Fracture for µG Exposure on Abscissa
- Interpretation
  - Purple Line Represents 95% Assurance that Fracture Risk is below Line
  - E.g., at 300 Days of µG Exposure, 95% Certain, *based solely on the Data*, that Fracture Risk is <3%
Alternate Display Concept

- **Purpose:** Compare for Varying Mission Durations
  - Typical ISS – 180 Days
  - Proposed Extended ISS – 365 Days
  - Mars Missions
    - Outbound and Inbound – 180 or 270 Days
    - Surface Phase – 540 Days (18 Months)
    - Total Missions of 900 and 1080 Days
- **Bandaid Charts**
  - From 5th Quantile to 95th
  - Color Density Reflects Probability Density
  - Median Line
Astronaut Spaceflight Bone Fracture Risk

- Logarithmic Scale
- 5th, 50th, and 95th Quantile Contours
- Bandaids Superimposed on Contours

Truncated at $10^{-8}$ for More Detail
ISS Missions

Bone Fracture Risk

In-flight Bone Fracture Risk, 180 vs 365 Day Missions

Probability

1 e-20 1 e-18 1 e-12 1 e-8 1 e-4 1 e+00

365 Day Mission

180 Day Mission

In-flight Bone Fracture Risk, 180 vs 365 Day Missions

Probability

1 e-8 1 e-08 1 e-04 1 e-02
Astronaut Spaceflight Bone Fracture Risk Assessment Results

• Answers Question of Human Spaceflight Fracture Risk, *based solely on Available Data*, 95% Certain less than 3% out to 300 Day Exposures

• Provides Quantified Risk Assessments that are Believable, Repeatable, and Sound
Case Study 4: Space Shuttle Locker Door Failure Risk Assessment
**Locker Door Loose Screw Risk Assessment**

- **Problem:** Screws Holding Locker Door in Place in Shuttle Bay are Too Short
  - If Door Loses Integrity, or Falls off, Something could Penetrate the Shuttle Hull during Launch or Descent
  - What is The Risk of having a Loose Screw, that could then Lead to a Risk of Losing a Door

- **Decision:**
  - Replace and Retighten All Screws, *OR*
  - Delay Flight
Loose Screw Quantitative Risk Assessment Setup

- No Clear Decision Rule
  - Decision Discriminator: Risk of Losing RSR Door Integrity due to Losing Screws
  - However, Failure Modes (number of lost screws leading to lost Door) not Defined
  - No Set Acceptable Risk, but very Low
  - No Set Assurance for Acceptable Risk
- Use Simple Binomial Model for Probability of losing Screws and for Probability of losing Doors as a Result of Losing Screws
- Data – on the Latch/Hinge Plates
  - 8 Screws Observed Loose of 287 Tested
  - No Screws Lost (0), of Loose or Tight Screws
More Setup

• Simple Posterior for Risk of Loose Screw
  • Notation
    • $\theta \equiv P(\text{loose screw})$ – Risk of Screw becoming Loose
    • $\lambda \equiv \text{Number Observed Loose in Test (Datum)}$
    • $\nu \equiv \text{Number Tested (Datum)}$
  • Objective Prior: $\theta^{-\frac{1}{2}}(1-\theta)^{-\frac{1}{2}}$
  • Nice Analytical Posterior given the Data:

$$pd(\theta | \lambda, \nu) = \frac{\Gamma(\nu + 1)}{\Gamma(\lambda + 1) \Gamma(\nu - \lambda + 1)} \theta^{\lambda} (1-\theta)^{\nu - \lambda} \ast \theta^{-\frac{1}{2}} (1-\theta)^{-\frac{1}{2}}$$

$$\propto \theta^{\lambda - \frac{1}{2}} (1-\theta)^{\nu - \lambda - \frac{1}{2}} \cdot Beta\left(\theta | \lambda + \frac{1}{2}, \nu - \lambda + \frac{1}{2}\right)$$
Latch/Hinge Plate

Loose Screw PRA Results

• Data
  • 8 Screws Observed Loose in Test
  • 287 Screws Tested

• Results Synopsis
  • Mode (Most Likely Risk) at 2.62% Risk
  • Mean: 2.95% Risk
  • 25th Percentile: 2.23% Risk
  • 75th Percentile: 3.55% Risk
Latch/Hinge Plate Loose Screw PRA Bandaid Chart

- Legend:
  - Range from 5th to 95th Quantiles – 1.52% to 4.75% Risks
  - Black Line: Median – 2.84% Risk
  - Dashed Line: Mean – 2.95% Risk
  - Mode: Solid Grey Line – 2.62% Risk
  - Dotted Lines: 25th and 75th Quantiles – 2.33% and 3.55% Risks Respectively
Risk of Loss of a Screw, Given that it is Loose

- What is the probability $\Lambda_L$ that a screw will be lost, given that it has become loose, based on our data?
  - We Have Relevant Data: 0 screws were lost of the 8 that were loose
  - This has a Nice Analytical PRA Solution:
    $P(\Lambda_L|\lambda, \nu) \sim \text{Beta}(\Lambda_L | \frac{1}{2}, \lambda + \frac{1}{2})$
Risk of Loss of a Screw, Given that it is Tight

- What is the probability \( \Lambda_T \) that a screw will be lost, given that it is has acceptable Back Away Torque, based on our data?
  - We Have relevant Data: 0 screws were lost of the 279 that did not have unacceptable Back Away Torque
  - This has a Nice Analytical PRA Solution:
    \[
    P(\Lambda_T|\lambda, \nu) = \text{Beta}(\Lambda_T| \frac{1}{2}, \nu - \lambda + \frac{1}{2})
    \]
Predicted Risks

- Risk Assessment Allows Computation of Predictive Distributions for Risks for the Future, Based on Existing Data
  - Rarely Used, But Usually the Seminal Issue (true here)
  - Usually Difficult to Compute, But Obtained Nice Analytical Solutions for This Problem
- For any $m$ Screws Loose in a Pattern of $n$ Screws:
  \[ P(m|\lambda, \nu, n) \sim \text{BetaBinomial}(m|\lambda^{+\frac{1}{2}}, \nu^{+\frac{1}{2}}, n) \]
- For any $\mu$ Screws Tight in a Pattern of $\sigma$ Screws:
  \[ P(\mu|\lambda, \nu, \sigma) \sim \text{BetaBinomial}(\mu|\nu^{-\frac{1}{2}}, \lambda^{+\frac{1}{2}}, \sigma) \]
- For any $M$ Screws Lost in a Pattern of $N$ Screws:
  \[ P(M|\lambda, \nu, N) \sim \text{BetaBinomial}(M|\frac{1}{2}, \lambda^{+\frac{1}{2}}, N) \]
Risk of Panel Door Loss

• Complex Risk Question
  • Loss of any Latch or Hinge Plate on Door will cause Loss of Door Integrity
  • Loss of a Latch or Hinge Plate requires Loss of One or More Screws
  • How many lost screws, in what patterns for Latch or Hinge Plate will Cause Loss of Door?
  • The Answer Defines Failure Modes

• Potential Failure Modes
  • Any One to Six Screws Lost in a Latch or a Hinge Plate Causes Door Integrity Loss - Conservative
  • Specific Pattern of One to Six Screws Lost in a Latch or Hinge Plate Causes Door Integrity Loss – Realistic Engineering, and Less Conservative
The Probability Equations for Risk of Panel Door Loss

- The Complete Probability Equations are usually Neglected, Usually a Mistake
- The Probability Statements for this Risk
  - \( P(\text{loss of any door}) \)
    \[
    = 1 - (1 - P(\text{loss of single panel door}))^{(# \text{ of single panel doors})} \times \left(1 - P(\text{loss of double panel door}))^{(# \text{ of double panel doors})} \times \left(1 - P(\text{loss of triple panel door}))^{(# \text{ of triple panel doors})}\right)
    \]
  - \( P(\text{loss of door}) \)
    \[
    = P(\text{loss of any Latch OR loss of any Hinge Plate on the door})
    = 1 - (1 - P(\text{loss of latch}))^{(# \text{ of latches and hinge plates on door})}
    \]
  - \( P(\text{loss of latch}) = P(\text{loss of Hinge Plate}) \)
    \[
    = P(\text{M screws lost of Pattern of 6}) \text{ – the failure mode}
    = \sum_{j=0}^{6} \left[ P(\text{M Lost | } j \text{ Loose}) \times P( j \text{ Loose}) + P(\text{M Lost | 6 - } j \text{ Tight}) \times P(6 - j \text{ Tight}) \right]
    \]
Predicted Risk of RSR Panel Door Failure

- Consider All Conservative Failure Modes (1 to 6 screws may be needed to Retain Each Latch and Each Hinge Plate)
- Worst Case – Specific Screw Patterns will Reduce Risk
- Table of Predicted Risks for Failure due to Lost Screws

| Failure Mode Definition (# Lost Screws in Pattern of 6) | P(Loss Single Door|Data) | P(Loss Double Door|Data) | P(Loss Triple Door|Data) | P(Loss Any Door|Data) |
|--------------------------------------------------------|----------------|----------------|----------------|----------------|----------------|
| 1 or more                                              | 1.91%          | 3.78%          | 5.62%          | 29.34%         |
| 2 or more                                              | 2.35e-2%       | 4.69e-2%       | 7.04e-2%       | 0.422%         |
| 3 or more                                              | 2.57e-4%       | 5.14e-4%       | 7.71e-4        | 4.63e-3%       |
| 4 or more                                              | 2.23e-6%       | 4.47e-6%       | 6.70e-6%       | 4.02e-5%       |
| 5 or more                                              | 1.34e-8%       | 2.68e-8%       | 4.02e-8%       | 2.41e-7%       |
| 6                                                      | 4.11e-11%      | 8.23e-11%      | 1.23e-10%      | 7.41e-10%      |
Summary and Conclusions

• Quantitative Risk Assessments for Any Problem in SE are Now Possible
  • Can Use Decision Theoretic Bayesian Approach
  • Can Avoid All Assumptions and Over conservatism
  • Can Use All Data and Information, Including Outliers
  • Only Need to Write Out Formulae and Code MCMC – No Math Solutions Needed
  • All Case Study Examples Done in Less than 4 Hours Each with Less than a Page of Code Each
  • Quantitative Risk Assessments Agree with Decision Maker Heuristics

• Properly Done Quantitative Risk Assessments Lead to Better Decisions, and Much More Cost Effective Risk Management
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References